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Retaining Connectivity in Multi-Task Communications Network with Multiple Agents: Connectability Theory Approach

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***Abstract*— Practical retention of mobile ad hoc network communications via connectability theory is presented and compared to predictive modeling techniques. Network communication disruptions is prevented by driving relay agents to computed waypoints using sliding mode and LQ control, or using predictive modeling to optimally control relay agents. The connectability matrix is used to determine where future node isolation will occur. This paper expands the connectability matrix concept into connectability theory to not only predict node isolation, but to directly compute the waypoints for relay agents. The existing methods of computing waypoints, of controlling robotic routers to form so called network bridges, and the outcome of predictive modeling are shown to be special cases of the proposed connectability theory. Also, case studies and simulations are presented to show this connectability theory's utility in various network configurations.**

I. Introduction

A primary mission agent (PA) is an agent that performs an assigned mission without regard for maintaining network communications. PAs may be controlled autonomously, by another system, or by a human operator. These agents may cooperate with other agents or systems in order to accomplish their assigned task, but they are considered uncooperative from the standpoint of preventing network disruptions. The PAs perform their assigned tasks without regard for assuring network connectivity, while relay agents (RAs) are employed to prevent PA isolations. Preserving connectivity among these PAs with only measurements of their positions was considered in previous work [1]-[3].

A. *Overall Problem Description*

Traditional graph theory [1], predictive modeling [3], the Fiedler eigenvalue maximization [3], a hybrid system approach [2], LQR [2], [5] and sliding mode control and observation techniques [1], [3] have been shown to provide solutions for the task of preservation of the network connectivity. The traditional graph theory approach [1] works when only one PA is assumed to become isolated on any given time window. Like the traditional graph theory approach, predictive modeling relies on calculating eigenvalues of the graph Laplacian in order to predict loss of connectivity, but no additional information is provided via the eigenvalues. Neither the predicted isolated agents nor the predicted broken network links are readily provided by traditional spectral graph methods.

The math model for the system is

$$\begin{cases}\dot q = A_1 q + g(t)\\ \dot z = A_2 z + Bu\end{cases} \quad where \quad \begin{cases}\dot q_i = p_i\\ \dot p_i = g_i(t)\end{cases} \begin{cases}\dot z_k = w_k\\ \dot w_k = u_k\end{cases} \qquad (1.1)$$
$$i = 1,\dots,n,\ k = 1,\dots,m$$

where $q_i, p_i \in \mathbb{R}^2$ are the position and velocity of each PA and $z_k, w_k \in \mathbb{R}^2$ are the position and velocity of each RA. The function $g(t)$ is the external commands and disturbances to the PAs and may be unknown by the RA controllers. The waypoint $z_{k+1}(t_j)$ of RA $k+1$ at $t = t_j$ is computed based on the prediction of discontinuity as described in Section II.

The algorithm from the previous work of predictive modeling [3] consists of the following components:

1. A module to estimate the unknown PA inputs in finite time;
2. A module to check connectivity among agents at regular intervals of time t_k;
3. A module to predict the loss of connectivity and to make a decision to deploy new RAs if required;
4. (a) An algorithm to create control policies for the RAs to ensure connectivity over a short time window $[t_0, t_0 + N\tau)$ based on sampling times τ.
   (b) An algorithm to compute the waypoints for the RAs to create a connected phantom graph at $t_0 + N\tau$, and a module to drive the RAs the predicted waypoints before the node isolation occurs.

Module 1 is implemented [1], [2] in a format of higher order sliding mode (HOSM) observer that estimates the velocity, p_i, as:

$$\begin{aligned}\dot\zeta_0 &= v_0\\ v_0 &= -3L^{1/3}\left|\zeta_0 - q_i\right|^{2/3} sign(\zeta_0 - q_i) + \zeta_1\\ \dot\zeta_1 &= v_1\\ v_1 &= -1.5L^{1/2}\left|\zeta_1 - v_0\right|^{1/2} sign(\zeta_1 - v_0) + \zeta_2\\ \dot\zeta_2 &= -1.1L sign(\zeta_2 - v_1)\end{aligned} \qquad (1.2)$$

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where L is a Lipchitz constant such that $L > \max\|g_i(t)\|$ [5]. The HOSM observer in equation (1.2) converges to the estimated PA velocity $\zeta_1 \to p_i$ and acceleration $\zeta_2 \to g_i(t)$ in finite time. The estimated velocity and reconstructed acceleration are used to calculate the position q_i on each interval (t_k, t_{k+1}).

Module 2 that checks the network connectivity and predicts the loss of connectivity in traditional graph theory, and predictive modeling both utilize the method of computing the eigenvalues of the graph Laplacian using the predicted values of the PAs from equation (1.2) [1], [2]. The agents are considered to move in the $\mathbb{R}^2$ plane. The interconnection of adjacent agents is modeled as an undirected graph $G_n:(V_n,\Omega_n)$ where $V_n:\{1,2,...,n\}$ is a finite set of vertices consisting of n agents and $\Omega_n \subseteq \{(i,j): i,j \subset V_n, i \neq j\}$ is the edges of G which are the communication links between agents. The graph is considered undirected because the existence of an edge between nodes means that bidirectional communications are insured. The distance between two adjacent nodes i and j is given b

$$\|q_i - q_j\| = r_{ij} = r_{ji} > 0 \tag{1.3}$$

where $q \in \mathbb{R}^2$, and $r \in \mathbb{R}$.

Definition 1. The ***distance matrix***, $D(G)$, of graph G is a symmetric matrix containing the distance between each node in graph G [6],[7]. $D(G)$ is given by:

$$D = \left[D\right]_{ij} = r_{ij} \quad i,j \in V_n \tag{1.4}$$

Definition 2. The ***adjacency matrix***, $A(G)$, of graph G is a symmetric matrix containing the adjacency relationships of each node in graph G [6] ,[7]. $A(G)$ is computed as:

$$A_{ij}(G) = \begin{cases} 1 & if\ r_{ij} \leq d^*, i \neq j \\ 0 & if\ r_{ij} > d^*, or, i = j \end{cases} \tag{1.5}$$

On each time window, a state dependent graph $G(t_k)$ is constructed based on the instantaneous position measurements of the agents present at time t_k, which become the nodes of the graph $G(t_k)$ [10]. Associated with $G(t_k)$, a state dependent Laplacian matrix

$$\left|\mathcal{L}_{G_{t_k}}\right|_{ij} = \begin{cases} -v_{ij}(t_k) & \text{if } i \neq j \\ \sum_{k \neq i} v_{ik}(t_k) & \text{if } i = j \end{cases} \tag{1.6}$$

where $v_{ij}(t_k)$ is the weight associated with each edge $V_{ij}(t)$. The $v_{ij}(t_k)$ are essentially the entries of the state dependent adjacency matrix $[A_{G_t}]_{ij}$ when $i \neq j$. The value of the $v_{ij}(t_k)$ depends on the distance between the i^{th} and j^{th} node and is defined as

$$v_{ij} = \begin{cases} 1 & \|r_{ij}(t)\| < d^* \\ \epsilon^{(d^* - d_{ij}(t))/(d^* - R)} & d^* \leq \|r_{ij}(t)\| \leq R \\ 0 & \|r_{ij}(t)\| > R \end{cases} \tag{1.7}$$

where ϵ, d^*, and R are the positive scalars and define the decay of the proximity strength between agents.

At time $t_k = t_0$, suppose there are *n* PAs and *m* RAs in the network. The state dependent graph $G(t_0)$ consists of $n+m$ nodes. At $t_k = t_0$, $r_{ij}(t_0)$, $v_{ij}(t_0)$ and the Laplacian matrix $\mathcal{L}_{G(t_0)} \in \mathbb{R}^{nxn}$ are evaluated. Since the Laplacian matrix of a state dependent graph is always symmetric positive semi-definite, the eigenvalues of $\mathcal{L}_{G(t_0)}$ are all non-negative [10]. Order the spectrum as

$$\lambda_1(\mathcal{L}_{G_0}) = 0 \leq \lambda_2(\mathcal{L}_{G_0}) \leq \ldots \leq \lambda_{n+m}(\mathcal{L}_{G_0}) \tag{1.8}$$

so that $\lambda_2(\mathcal{L}_{G_0})$ represents the second smallest eigenvalue associated with the algebraic connectivity of the graph [3]. By its formulation, the term $\lambda_2(\mathcal{L}_{G_0})$ represents a measure of how connected the agents are on the two-dimensional plane based on the chosen proximity rule. The term $\lambda_2(\mathcal{L}_{G_0})$ becomes zero when G_0 becomes disconnected, and the value increases when the graph is tightly connected [3]. Provided all the agents are connected and there exists at least one spanning tree; the rank of the Laplacian of G_0 is $n-1$.

Module 3 employs the traditional graph theory as these and predictive modeling algorithms diverge after loss of network connectivity is predicted. Traditional graph theory based methods [1], [2] assume only one PA is isolated on a time interval. The adjacency matrix just before the predicted isolation is compared to that matrix at the time of the isolation to find which agent is isolated. The waypoint is computed as the midpoint between the isolated PA and its nearest neighbor. The new RA is then driven to the waypoint via sliding mode control along a computed path.

Module 4 is two-fold:

The first approach is based on predictive modeling, when all RAs are engaged on each time interval, and their control, $\{u^{i*}(t_k)\}_{k=0}^{N}$, is computed by minimizing a finite time horizon objective

$$\{u^{i*}(t_k)\}_{k=0}^{N} := \arg \min_{\{u^{i*}(t_k)\}_{k=0}^{N}} J(\{z^i(t_k)\}_{k=0}^{N}, \{\hat{g}^i(t_0)\}, \{u^{i*}(t_k)\}_{k=0}^{N})$$

subject to

$$
\begin{aligned}
& q^i(t_{k+1}) = Aq^i(t_k) + B\hat{g}^i(t_k),\ \forall i = 1,\dots,n \\
& z^i(t_{k+1}) = Az^i(t_k) + Bu^i(t_k),\ \forall i = 1,\dots,m \\
& \left\| r_{ij}(t_k) \right\|^2 \geq \left(d^*\right)^2,\ \forall i \neq j \text{ and } \therefore \forall k = \{1,\dots,N\} \\
& u_{\min} \leq u^i(t_k) \leq u_{\max},\ \forall i = \{1,\dots,N\}
\end{aligned} \tag{1.9}
$$

In [10] the finite horizon cost function J is defined as

$$
J(.) := \sum_{k=0}^{N} \left[\frac{\alpha_i}{\lambda_2(\mathcal{L}_{G_{t_k}}) + \delta} + \alpha_2 \sum_{i=1}^{m} \left\| u^i(t_k) \right\|^2 \right] \tag{1.10}
$$

where α_1 and α_2 are tuning weights [3].

The second approach is based on hybrid dynamical systems. On each time window, additional RAs added to the network increase the network state dimension. If already dispatched RAs can be repositioned and preserve connectivity, their new waypoints are computed and the RAs are commanded to the new waypoints. If these already dispatched RAs are insufficient and there are idle RAs available, waypoints for the necessary RAs are computed and those previously idle RAs are commanded to the waypoints. If previously idle RAs are invoked, the dimension of the system state increases. The condition that causes this increase in state dimension is, in hybrid system theory, called a *guard* condition. If already dispatched RAs are utilized, the dimension of the state remains the same at in the next time window, but the control changes, also representing a *guard* condition. In predictive modeling, the control in the right hand side of equations (1.9) and (1.10) also changes. Even though the system on each time window may be stable, the hybrid system stability across time window transitions must be assured. The assurance of system stability across jumps in state is provided by Lyapunov theory. Particularly, the theorem presented in [13] is used to assess the hybrid system stability in our case.

B. Overall Challenge

Mobile ad hoc networks (MANETs) are a popular solution for providing communications and data sharing over a relatively small geographic location particularly for sharing sensor data. Maintaining connections to deployed sensors is currently a topic of much research in the robotics community. Most such research in the robotics field emphasizes maximizing the useful life of remote battery powered sensors [12] via data harvesting robots or careful placement of mobile data routers. These remote sensors apparently are used for monitoring and are placed in carefully measured grids since many papers focus on or refer to coverage of a plane by circles and ratios of sensing distance to transmission distance. These generally are not mobile sensors. Research in the robotics field on MANETs has also provided solutions to building and maintaining a communications bridge of mobile robotic routers [4] to connect a mobile agent to a static base station. It is shown in this paper that this concept of a communications bridge is a simple case of connectability theory. The prediction of loss of connectivity and computing the waypoints for RAs is still a challenge and requires a thoughtful study and improvement. The connectability matrix has been already used [2] to determine where future node isolation would occur. This paper expands the connectability matrix concept into connectability theory to not only predict node isolation, but to directly compute the waypoints for relay agents. The existing methods of computing waypoints, of controlling robotic routers to form so called network bridges, and the outcome of predictive modeling are shown to be special cases of the proposed connectability theory.

C. Contribution Statement

The main contribution of this paper is to present and use connectability theory to predict node isolation, identify which nodes are affected, and compute where relay nodes should be placed to preserve connectivity so that the relay nodes may be driven to those waypoints by control algorithms. Connectability theory is an extension of the connectability matrix that was introduced in [2], and can be used to directly determine which agents are isolated, how many controlled relay agents (RAs) are required to prevent the isolations, and where those RA waypoints should be dropped to create a phantom graph that preserves the network communications.

The paper is organized as follows. Section II expands the connectability matrix introduced in [2] into connectability theory. Application of connectability theory for select cases of interesting PA configurations is presented in Section III. Section IV contains a simulation example using connectability theory. Section V concludes the paper.

II. Connectability Theory Defined

A. Connectability Matrix of Graph

The connectability matrix was introduced in [2] as a means to predict agent isolation.

Definition 3. The ***connectability matrix***, $C(G)$ of G is the symmetric matrix consisting of integer values that are the number of intervals of size d^* separating nodes i and j given by equation (2.1) [2]:

$$
[C]_{ij} = \begin{cases} k = ceil([D]_{ij} / d^*) & if\ i \neq j \\ 0 & if\ i = j \end{cases} \tag{2.1}
$$

B. Connectability Metric of Graph

The connectability metric, m_1, was mentioned in [2] as a means to easily identify isolated nodes. Here the connectability metric is expanded.

Definition 4. The ***connectability metric***, $m_k = col[m_k^i]$, of graph G is a column matrix where m_k is computed as:

$$
m_k = \left[m_k^i = \begin{cases} l & where\ l = count\left([C]_{i,j} = k\right) > 0 \\ 0 & if\ [C]_{i,j} \neq k\ \forall j = 1,2,\dots,n \end{cases} \right] \tag{2.2}
$$

where $k = 1,2,...,\max\left[C\right]$. The graph $\kappa_v - connectivity$ [6],[14] of traditional graph theory is directly identifiable using the m_1 connectability metric since $\kappa_v = \min_i(m_k^i)$.

Properties of the ***connectability metric*** are:

Property 1. If the m_1 metric elements are greater than zero the following conditions hold:

a. If $\max(m_1) = 1$, there are only pairs of connected nodes.
b. Elements of $m_1 > 1$ represent multiply connected nodes.
c. If $count(m_1 = 1)$ > $count(m_1 > 1)$, there are disconnected clusters of nodes.
d. If all entries $m_1 = 2$, then the network is an n-cycle graph and is 2-edge connected. [14]
e. If $count(m_1 \geq 2) \geq n-2$, then all nodes are connected in a tree.

Property 2. The connectability metrics defined in (2.2) satisfy

$$\sum_{i=1}^{n} m_1^i = 2k,\ k = 1,2,...,n\cdot(n+1) \tag{2.3}$$

which means there can only be an even number of connected nodes. This property is related to Harary graphs. [14]

Property 3. If $m_i \neq [\emptyset], m_{i+1} = [\emptyset],\ m_{i+2} = [\emptyset], ..., m_{i+k} \neq [\emptyset]$ for any $i, k > i+1$, then there are clusters of nodes separated by $k-1$ intervals.

C. Connectability Indices of Graph

The connectability indices, $indxC_k^i$, for each row $i = 1,...,n$ of C contains the indices of C that are equal to $k = 1,...,\max\left[C\right]$. The connectability indices, $indxC_k^i$ are computed as:

$$indxC_k^i = \begin{bmatrix} j & if\ \left[C\right]_{ij} = k \\ 0 & if\ \left[C\right]_{ij} \neq k \end{bmatrix} \tag{2.4}$$

where $i, j = 1,2,...,n$.

Properties of the ***connectability indices*** are:

Property 1. If any elements of $indxC_k^i > 0$, the m_k^i metric elements are the number of non-zero $indxC_k^i$ elements.

Property 2. The non-zero entries of $indxC_1^i$ are the nodes connected with node i.

Property 3. The non-zero $indxC_k^i$ entries are the indices of the nodes that are **k-connectable** to node i.

Theorem: The centroid of

intersection ($indxC_2^i, indxC_2^{i+1}, \ldots, indxC_2^{i+p}$)

is the location to insert an RA waypoint to maintain communications between the nodes.

Proof: From connectability indices property 3, $indxC_2^i$ contains the indices of the nodes that are 2-connectable or can be connected with one relay node. For different nodes i, ..., $i+p$,

intersection ($indxC_2^i, indxC_2^{i+1}, \ldots, indxC_2^{i+p}$)

yields the set of nodes that are 2-connectable to both nodes i , …, $i+p$. Therefore, computing the centroid of the set of nodes gives the (x, y) location common to the set.

III. Connectability and Waypoint Calculation

For a system to be implemented, connectability assessment and necessary waypoint computation must be performed. Also, path planning and control methods must be implemented to drive the RAs to the computed waypoints. In our previous work, sliding mode control [1] and hybrid LQR control [2] were implemented to perform the control function. In these examples, connectability theory provides the connectability assessment and waypoint computation.

A. Network Bridge of Mobile Routers

In [4], a wireless network bridge of robotic routers was created to connect a mobile agent to a fixed base. The wireless bridge examples provided in [4] assume that the network is disconnected. This example demonstrates that such wireless network bridges are a specific application of connectability theory. In addition, the m_1 metric is used to determine the connectivity of the network and the m_k metric indicates how many relay nodes must be inserted. Figure 1 shows the initial configuration of the mobile agent (agent 1) and the fixed base (agent 2). The distance between the mobile agent and the base is 424.26 m and the maximum communications distance is 100 m. The red circles indicate that the agent is disconnected from the base.

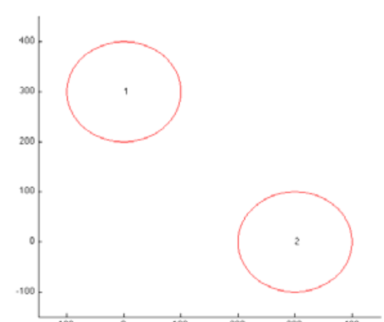

Figure 1. Initial configuration of mobile agent and fixed base

Since this case is simplistic, the connectability matrix, metric and indices are also quite simplistic. The connectability matrix, metric and index for the initial configuration are shown in Table 1. The connectability matrix indicates that the agent and the base are separated by $5 \cdot d^*$. The m_5 metric shows that the agent and the base are connectable by 4 RAs, and $indxC_5$ shows that agent 1 is connectable to agent 2 and vice versa.

Cmatrix	1	2
1	0	5
2	5	0

m1	m2	m3	m4	m5
0	0	0	0	1
0	0	0	0	1

indxC5	1	2
1	2	
2	1	

Table 1. Initial connectability matrix, metric, and index

The waypoints for the four RAs are calculated by placing each waypoint d^* apart along the line between the agent and the base, agreeing with the method shown in [4]. Figure 2 shows the network bridge of mobile routers connecting the mobile agent and the fixed base. The blue circles outlining each agent's communication range indicate that the agents are connected.

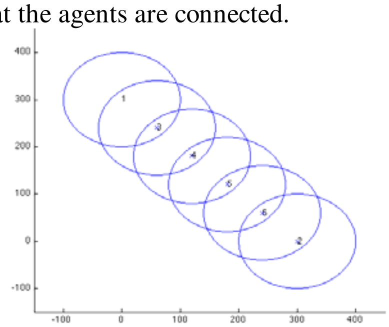

Figure 2. Network bridge connecting the mobile agent and fixed base.

Table 2 contains the final connectability matrix, metric, and index. The connectability matrix, metric and index in Table 2 show that the agents are connected. In particular each row of $indxC_1$ shows the interconnected agents.

Cmatrix	1	2	3	4	5	6
1	0	5	1	2	3	4
2	5	0	4	3	2	1
3	1	4	0	1	2	3
4	2	3	1	0	1	2
5	3	2	2	1	0	1
6	4	1	3	2	1	0

m1
1
1
2
2
2
2

indxC1	1	2
1	3	
2	6	
3	1	4
4	3	5
5	4	6
6	2	5

Table 2. Final connectability matrix, metric, index, and eigenvalues

B. *Maximizing Fiedler Eigenvalue in Complex PA Pattern*

Another case study is the arrangement of PAs displayed in Figure 3. In this configuration, as indicated by the red and blue circles, agents 1, 3, and 5 are disconnected while agents 2, 4, and 6 are connected.

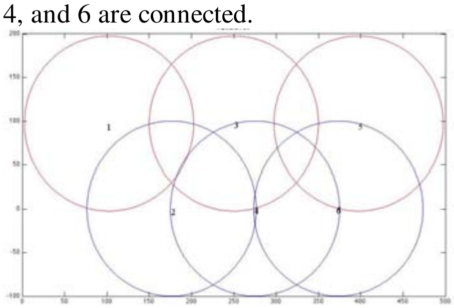

Figure 3. Complex PA configuration

The coordinates of the set of PAs is {(102.5,97.5), (176.45,0), (250,97.5), (275,0), (397.5,97.5), (373.55,0)}. The connectability matrix, metrics, and indices for this configuration are contained in Table 3. The elements of metric $m_1 = 0$ show that agents 1, 3, and 5 are disconnected, while the non-zero entries indicate the number of agents that are connected to the agent with ordinal value of the row number. The elements of $indxC_1$ show which agents are connected. The elements in $indxC_2$ shows which agents are connectable with one RA.

Cmatrix	1	2	3	4	5	6
1	0	2	2	2	3	3
2	2	0	2	1	3	2
3	2	2	0	2	2	2
4	2	1	2	0	2	1
5	3	3	2	2	0	2
6	3	2	2	1	2	0

m1	m2	m3
0	3	2
1	3	1
0	5	0
2	3	0
0	3	2
1	3	1

indxC2	1	2	3	4
1	2	3	4	
2	1	3	6	
3	1	2	4	5
4	1	3	5	
5	3	4	6	
6	2	3	5	

indxC1	1	2
1		
2	4	
3		
4	2	6
5		
6	4	

Table 3 . Initial connectability matrix, metrics, and indices

The waypoints are computed by using the values in $indxC_2$ as indices into the PAs. Finding the intersection of row 1 and 3, $indxC_2^1 \cap indxC_2^3 = \{1,2,3,4\}$, indicates that agents 1,2,3, and 4 are connectable by one RA, but since agents 2 and 4 are already connected, the waypoint is computed as $centroid(1,2,3) = (161.6,51.1)$. Likewise, the intersection of row 3 and 5, $indxC_2^3 \cap indxC_2^5 = \{3,4,5\}$, indicates that agents 3, 4, and 5 are also connectable by one RA. The waypoint is computed as $centroid(3,4,5) = (324,48.75)$. With RAs placed at these waypoints, Figure 4 shows that all agents are connected.

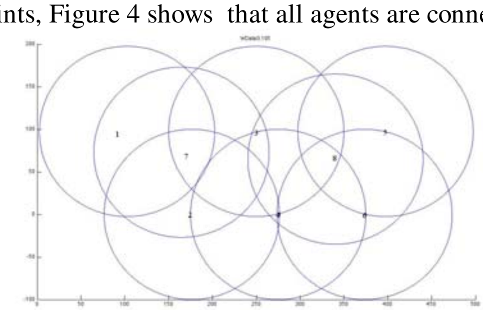

Figure 4. Final configuration of the phantom graph

Notice that dropping the waypoint (agent 7) at $centroid(1,2,3)$ does not connect with agent 4. This results in the graph Laplacian eigenvalues shown in Table 4. Since the Fiedler eigenvalue, $\lambda_2 > 0$, the network is connected.

Eigenvalues
0
0.496495501
1
1.585786438
1.735623407
3.576712516
4.414213562
5.191168576

Table 4. Eigenvalues of phantom graph's Laplacian

If the waypoint for agent 7 is instead dropped at $centroid(1,2,3,4)$ the eigenvalues in Table 5 show that the Fiedler eigenvalue is maximized. This shows that the predictive modeling method of maximizing the Fiedler eigenvalue is a special case of connectability theory.

Eigenvalues
0
0.622797146
1.082166755
1.726109445
1.756105965
4.334319501
4.651093409
5.827407778

Table 5. Maximized Fiedler eigenvalue of phantom graph's Laplacian

IV. Simulation

The scenario for preserving network connectivity using connectability theory is taken from the Section III-B. The plot of the agents' trajectories is shown in Figure 5. The blue lines are the PA trajectories and the red lines are the RAs. As the PAs move, the PAs' positions are used to estimate their velocity on each time window using equation (1.2). When network connectivity is predicted to be broken on a time window, the RAs are dispatched to the computed waypoints. Path planning and trajectory tracking [1] are used to create and follow a safe path for the RAs to follow. The RAs are driven to the waypoints using hybrid LQR control [2], or high order sliding mode control (HOSM) [1] if disturbances are present. When the two RAs are introduced to the network, the system's state dimension increases resulting in jumps as described in Section I.

The PAs' initial positions are: $\{(150,50)$, $(200,0)$, $(250,50)$, $(275,0)$, $(350,50)$, $(350,0)\}$ and the RAs' initial positions are $\{(0,0),(0,0)\}$. The PAs' final positions are $\{(91,109)$, $(170.5,0)$, $(250,109)$, $(275,0)$, $(409,109)$, $(379.5,0)\}$. The final RA positions are $\{(170.5,72.67)$, $(341.167,72.67)\}$. Movement of the PAs result in a graph that is configured similar to that of Figure 3.

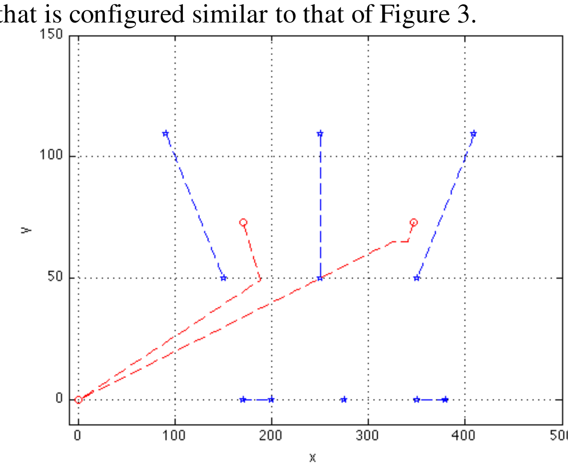

Figure 5. Plot of PA and RA locations from 0 to 120 seconds.

V. Conclusions

Connectability theory was defined and used to perform connectivity assessment and waypoint generation directly and easier than in previous work. A simulation was provided to demonstrate connectability theory in an integrated control system, utilizing hybrid LQR control. The problem as addressed by predicting the loss of connectivity and adding relay agents to preserve connectivity. The loss of connectivity was predicted via the m_1 connectability metric, and the two RAs' waypoints were computed using the $indxC_2$ connectability index

The utility of connectability theory was shown for two configurations of PAs. The first configuration corresponded to the communications bridge from [4]. This showed that the communications bridge is a simplistic case of connectability theory. In the second case, the waypoints were dropped at the centroid of multiple PA communication regions. It was shown that disregarding already connected PAs, the centroid calculation had reduced complexity but resulted in connection of the network. If all of the connectable PAs were used in the centroid calculation the resulting network was not only connected, but the Fiedler eigenvalue was maximized. In this example, the predictive modeling approach [3] was shown to be a special case of connectability theory.

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